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SEPARATING THE VARIANCES OF NOISE COMPONENTS IN THE GLOBAL POSITIONING SYSTEM

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ABSTRACT

Central to the success of the GPS program is the ability to model the frequency stability characteristics of the its various components. A persistent challenge in evaluating the Global Positioning System is the separation of the errors of the satellite clocks from those due to the satellite ephemeris errors and/or the signal propagation delay errors. This information is important when one tries to improve the performance of the Global Positioning System. It is necessary to know if a particular component of the system meets specification and which component(s) limits performance.

Although one cannot separate the errors themselves, a method has been developed whereby the "Allan variances" of critical components to the GPS can be separated. Using a reference clock such as UTC(NBS) or UTC(USNO), for example, the fractional frequency stability of each of the following can be separated from each of the others: the reference clock, the space vehicle clock, the GPS clock, the clock upload correction, the ephemeris and the propagation delay. This technique has the potential to significantly assist in properly setting the parameters to obtain optimum performance from the Global Positioning System e.g. setting the Kalman filter parameters. Results will be given showing some interesting surprises in the characteristics of the system.

INTRODUCTION

During the testing of the GPS it has become evident that an independent method for the characterization of the observables of the system would be an important supplement to the Kalman estimates. Such a method would allow one to diagnose problems, make improvements, and predict system performance for variation in the system's environment. This is not an easy task in some cases. For example, an independent method for the separation of the time errors of the clocks from those due to the ephemeris variations and propagation delays has been desired for some time. Because of the natural correlation between these error sources, differences of opinion have often arisen as to the source of some errors that have been observed.

Under a reasonable set of assumptions, NBS has developed a method whereby the Allan variances of important GPS observables can be separated. Using a reference clock such as UTC(NBS) or UTC(USNO), for example, the fractional stability of each of the following can be separated from each of the other:

- the reference clock
- the space vehicle clock
- the GPS clock
- the clock upload correction
- the ephermeris and the propagation delay

This technique has the potential to significantly assist in properly setting the GPS Kalman filter parameters. Also, this technique should be useful to users who want to study the stability of their clocks, since the user's reference is one of the components separated.

GENERAL CONSIDERATIONS

This separation of variances is performed using the following general approach. When considering any time series it is convenient to divide the elements into two parts, i.e., the deterministic part and the random part which is described by stochastic measures such as spectral densities or Allan variances. First the random elements are separated from the deterministic ones. If the driving forces that cause the random perturbations are independent, then the variances of these individual components can also be separated. An example is the separation of the sum of the ephmeris errors and propagation delay variations from the clock upload correction errors. In short term these appear to be correlated but in long term they decorrelate and hence the variances of these can be separated.[1] If different space vehicles are observed within a reasonable period of time, mainly within a few hours of each other, then the clock in each of the satellites provides an independent reference having random uncorrelated errors with the clocks in other satellites and with the ground clock. Using three independent satellites allows one to calculate variances for each individual component among the three. Finally, the clock correction error is independent of the time of the space vehicle clock, thus providing a tool for the separation of the variance of the errors in that component of the system. This approach and these assumptions will be explained further as the method is developed in detail.

Suppose that we have three independent time series -- denoted by subscripts i,j, and k. Since all measurements are made in pairs one can use those pairs of measurements to estimate the individual variances (σ_i^2, σ_j^2, and σ_k^2) of the time series from the variances of the pairs of measurements (σ_{ij}^2, σ_{ik}^2, and σ_{jk}^2). The variance of each time series is estimated as follows

$$\sigma_i^2 = (\sigma_{ij}^2 + \sigma_{ik}^2 - \sigma_{jk}^2)/2, \qquad (1)$$

and one permutes the i, j, and k to obtain σ_j^2 and σ_k^2. A problem which sometimes arises with this technique is that the estimated variances are negative. This may occur when there are too little data or there are apparent correlations. The longer the data length the better the statistical confidence on the estimates. For the GPS case one can use 3 independent satellites to estimate observables for each of the satellites in question.

DERIVATION FOR GPS

Next we will describe the individual independent error sources arising in a given GPS time transfer measurement. We will define some terms as follows: Let x be the time deviation for a particular noise source in the GPS. We will subscript the x depending upon the particular source being studied.

REF... as the reference clock such as the NBS clock.
GPS'...the received estimate of the time from the GPS receiver.
GPS ...time as generated by the GPS master clock.
PE ...the combination of the propagation time error and the satellite ephemeris error.
CL' ...the error in the space vehicle clock correction.
SV' ...the time of the space vehicle clock as received by the GPS receiver
SV ...the true SV time as generated within the space vehicle.

Our goal, of course, is to have an estimate of the true variance of each one of the components in question.

We may make the following measurements for each of 3 satellites i, j, and k; specifically we will list these for satellite i.

$$x_{REF-GPS'_i} = x_{REF} - x_{PE_i} - x_{CL'_i} - x_{GPS} \tag{2}$$

$$x_{REF-SV'_i} = x_{REF} - x_{PE_i} - x_{SV_i} \tag{3}$$

If equation (2) is subtracted from equation (3) one obtains cancellation of the propagation plus ephemeris error.

$$x_{GPS'_i-SV'_i} = x_{GPS} + x_{CL'_i} - x_{SV_i} \tag{4}$$

Assuming that the reference clock time deviation error is small from one satellite measurement to the next, which is a good assumption for high quality references with errors of the order of one nanosecond , one can subtract the measurements of satellite i from those of satellite j resulting in the following 2 equations:

$$x_{REF-GPS'_{ij}} = x_{PE_{ij}} + x_{CL'_{ij}} \tag{5}$$

$$x_{REF-SV'_{ij}} = x_{PE_{ij}} + x_{SV_{ij}}, \tag{6}$$

where the ij subscripts on the right of equation (5) and (6) denote the differences (j-i) in the measurement of those two components. Equations (5) and (6) can, of course, be written for satellites i and k and for j and k as well. Equations 2 through 6 comprise our measurement basis. Since (in long term) each of the components in these equations are uncorrelated we may take variances of each of these equations and the cross terms will average to zero giving the following 5 equations:

$$\sigma^2_{1_i} \equiv \sigma^2_{REF-GPS'_i} = \sigma^2_{REF} + \sigma^2_{PE_i} + \sigma^2_{CL'_i} + \sigma^2_{GPS} \tag{7}$$

$$\sigma^2_{2_i} \equiv \sigma^2_{REF-SV'_i} = \sigma^2_{REF} + \sigma^2_{PE_i} + \sigma^2_{SV_i} \tag{8}$$

$$\sigma^2_{3_i} \equiv \sigma^2_{GPS'_i-SV'_i} = \sigma^2_{GPS} + \sigma^2_{CL'_i} + \sigma^2_{SV_i} \tag{9}$$

$$\sigma^2_{REF-GPS'_{ij}} = \sigma^2_{PE_{ij}} + \sigma^2_{CL'_{ij}} \tag{10}$$

$$\sigma^2_{REF-SV'_{ij}} = \sigma^2_{PE_{ij}} + \sigma^2_{SV_{ij}} \tag{11}$$

And again we can write equations (7), (8), and (9) for satellites j and k as well, and equation (10), (11), for satellite pairs ik, and jk as well. In addition we can use equation (1) to estimate the variances for i, j, and k separately from (10), and (11) respectively:

$$\sigma^2_{4_i} \equiv \sigma^2_{PE_i} + \sigma^2_{CL'_i} \tag{12}$$

$$\sigma^2_{5_i} \equiv \sigma^2_{PE_i} + \sigma^2_{SV_i} \tag{13}$$

and similarly for j and k. In matrix formulation we have the following representative set of equations resulting from the estimates or measures for each of the three satellites i, j and k:

$$\begin{pmatrix} \sigma^2_1 \\ \sigma^2_2 \\ \sigma^2_3 \\ \sigma^2_4 \\ \sigma^2_5 \end{pmatrix}_{i,j,k} = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \sigma^2_{REF} \\ \sigma^2_{GPS} \\ \sigma^2_{SV} \\ \sigma^2_{CL'} \\ \sigma^2_{PE} \end{pmatrix}_{i,j,k} \tag{14}$$

If the matrix is inverted we may then write the final equation for the variances of the individual components of the GPS system. Global variances are obtained through each of three space vehicles. Variances pertaining to an individual space vehicle are obtained through that space vehicle.

$$\begin{pmatrix} \sigma^2_{REF} \\ \sigma^2_{GPS} \\ \sigma^2_{SV} \\ \sigma^2_{CL'} \\ \sigma^2_{PE} \end{pmatrix}_{i,j,k} = \begin{pmatrix} 0 & 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & -1 & 1 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & -1 \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \sigma^2_1 \\ \sigma^2_2 \\ \sigma^2_3 \\ \sigma^2_4 \\ \sigma^2_5 \end{pmatrix}_{i,j,k} \qquad (15)$$

Alternatively, one can write out the separation of variances in equation form for each of the i, j, and k satellites -- here written specifically for i:

$$\sigma^2_{REF_i} = \sigma^2_{2_i} - \sigma^2_{5_i} \qquad (16)$$

$$\sigma^2_{GPS_i} = \sigma^2_{1_i} - \sigma^2_{2_i} - \sigma^2_{4_i} + \sigma^2_{5_i} \qquad (17)$$

$$\sigma^2_{SV_i} = \tfrac{1}{2}\,(-\sigma^2_{1_i} + \sigma^2_{2_i} + \sigma^2_{3_i}) \qquad (18)$$

$$\sigma^2_{CL'_i} = \tfrac{1}{2}\,(-\sigma^2_{1_i} + \sigma^2_{2_i} + \sigma^2_{3_i}) + \sigma^2_{4_i} - \sigma^2_{5_i} \qquad (19)$$

$$\sigma^2_{PE_i} = \tfrac{1}{2}\,(\sigma^2_{1_i} - \sigma^2_{2_i} - \sigma^2_{3_i}) + \sigma^2_{5_i} \qquad (20)$$

The subscript i on REF and on GPS, clocks which are totally independent of the satellites i, j, and k, denotes an estimate of that clock's stability calculated via that particular satellite. As a final answer one could take a simple average or a more sophisticated statistical (e.g., weighted) average of the $\sigma^2_{REF_i}$, $\sigma^2_{REF_j}$, and $\sigma^2_{REF_k}$, and of the $\sigma^2_{GPS_i}$, $\sigma^2_{GPS_j}$, and $\sigma^2_{GPS_k}$.

The Allan variance fulfills the criterion that the variance used be a well behaved stable measure of the time series in question. In fact one can do the Allan variance analysis for different sample times as well, which allows one to characterize a process. The only criterion on the sample time is that it be sufficiently long so that the processes under consideration are uncorrelated, indications are that this is of the order of one day and longer.

AN EXAMPLE

This theoretical approach was applied to data taken at NBS from June 25th, 1983 to October 1, 1983. The result was an analysis of the stability of the GPS and space vehicle clocks, as well as of the clock corrections and propagation noise plus ephermeris estimates.

The stability analysis was performed on data on file in the NBS time scale computer. Each file entry is characterized by the MJD on which the data was taken, the hour, minute and second on which the satellite pass was started, the receiver number used to receive the data, the space vehicle involved, the class byte employed, the length of the data, the age of the data, the elevation and azimuth of the satellite, the ionospheric delay, the reference minus space vehicle (SV) time to a tenth of a nanosecond resolution and its accompanying slope from a linear least squares fit to the particular data pass, the reference minus GPS time to a tenth of a nanosecond resolution and its similar accompanying slope from a linear least squares fit, and the rms fit of the linear least squares to that data set. In addition, for the NBS data, the time of the reference minus UTC(NBS) is recorded. Finally there is a column for any offsets which may be due to discontinuities from a known effect in a receiver or idiosyncrasy in the system. The NBS reference clock is typically within a few nanoseconds of UTC(NBS).

The most recent data were analyzed to give a current estimate of the stability of the clocks in the GPS. The period covered was from MJD 45510 to 45608, which is the 25th of June 1983 throughout the 1st of October 1983. The data are taken on a sidereal day basis; i.e., the receivers automatically subtract 4 minutes a day to nominally maintain the same viewing angle to the SV. On the 30th of September 1983 the starts of each track time were 1802 UT, 1855 UT, 2050 UT, 2228 UT for SV-8, -6, and -9 and -5 respectively. The track lengths were each 780 seconds. The average age of data over the analysis period was nominally 2 to 4 hours and the elevation angle in all cases was above 52 degrees.

All of the time difference plots have nanoseconds as their ordinate units and the abscissas are in MJDs. Figure 1 is a plot of UTC(USNO-MC) minus the time of the GPS steered clock sometimes called the GPS software clock. In short term this clock behaves like the cesium at Vandenburg or Alaska and in long term it should reflect the stability of UTC(USNO-MC) as the software clock is steered to UTC(USNO-MC). Only SV-8 data were used because of some problems in the other space vehicle data sets in the USNO file stored in the NBS time scale computer. One sees a fairly significant frequency step around MJD 45591 and the time at the end of September has departed more than one microsecond from UTC(USNO).

Figure 2 is the average obtained by SV-5, -6, -8 and -9 of the NBS reference (Clock 9) minus GPS steered clock. Clock 9 is kept within a few nanoseconds of UTC(NBS). One sees very similar performance between figures 1 and 2 indicating that the main effect is that of the GPS clock. The data in Figure 2 are smoother simply because of the average across additional satellites and because of somewhat quieter receiver data. The agreement between the 4 space vehicles of the time difference, Clock 9 minus GPS, was typically in the range of 6 to 8 nanoseconds.

Figure 3 is the plot of NBS Clock 9 minus Space Vehicle Cesium 5 with a frequency removed of 1.66 parts in 10^{12}.

Figure 4 is NBS Clock 9 minus the SV-6 Rubidium with a mean frequency removed of 7.009 parts in 10^{11}. The frequency drift of the rubidium has clearly changed from negative to positive from the first and middle of the data to the last part.

Figure 5 shows the NBS Clock 9 minus SV-8 with the time reset occurring at MJD 45573. The stability analysis below was reported after the reset to take advantage of the most recent stability information.

Figure 6 is NBS Clock 9 minus SV-9 and shows some measurable frequency drift in the NAVSTAR 6 Cesium. This could be problematic as frequency drift in cesium standards is sometimes indicative of end of life.

Figure 7 is NBS Clock 9 or SV-9 after removing a mean frequency drift by using a linear least squares fit and a mean time from the data. The mean frequency removed was .3858 x 10^{-11}. The drift was -.197 x 10^{-14}/day and the mean time removed was 9498 nanoseconds. If a stability analysis is performed on the residuals shown in Figure 7, one obtains the sigma tau plot shown in Figure 8 which indicates that the frequency drift is well modelled and the residual random instabilities are very small--of the order of 3 to 5 parts in 10^{14} for sample times of 4 to 32 days.

The last set of figures are $\sigma_y(\tau)$ plots as estimated from the separation of variance analysis technique. As shown earlier this technique allows one to estimate the contribution of the individual noise components to the stability. Figure 9 is the frequency stability of the GPS steered clock which appears to be of the order of 1 part in 10^{13}. An interesting phenomenon is observed in the long term, namely that the $\sigma_y(\tau)$ values tend to decrease which is indicative of the long term steering of the GPS time; the time constant appears to be of the order of a few weeks.

Figure 10 is a stability plot of the Space Vehicle 5 cesium and of the clock correction errors and of the ephemeris plus propagation errors. Because the measurements were made only a few hours after upload, the clock correction errors should be approximately one-tenth of those of the space vehicle clock errors. It is evident from this data that the clock correction errors are of the same order as the space vehicle instabilities. One possible explanation of this is that the Q value in the Kalman processor at Vandenburg is set so as to assign too much error to the SV clock and not enough to the ephemeris and propagation. One also sees from the same figure that the ephemeris and propagation errors are clearly well below those of the space vehicle clock.

Because of the three corner hat analysis technique used in the separation of variances routine and because of finite data sets, it is possible to have negative variances. A negative variance indicates that the noise level is well below the other components in the calculation. Given the data length involved in this data set one can at best resolve sigmas that are about one-tenth that of the other sigmas being considered in the analysis. In those cases where small or negative variances occurred they were simply not plotted. Thus the interpretation for those cases where no data points are plotted is that the stability of that component is well below that of the other components for those sample times. Figure 11 is a plot for NAVSTAR 3 (Space Vehicle 6). In this case, the propagation and ephemeris errors were significantly below either the space vehicle stability or the clock correction errors. Here again it seems that too much error is being assigned to the SV clock.

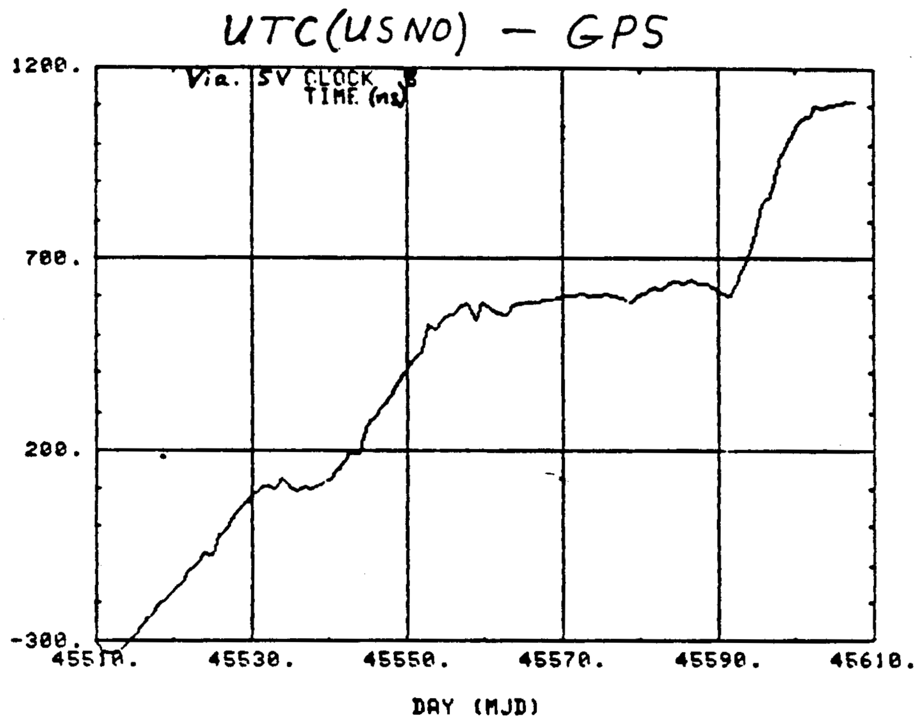
Figure 12 is a stability plot of Space Vehicle 8. Here again it appears that at τ = 1 day too much error is being assigned to the clock and not enough to the ephermeris and propagation. In long term however, that appears not to be the case and the clock correction error falls below that of the ephemeris plus propagation errors as well as the clock instabilities as it should.

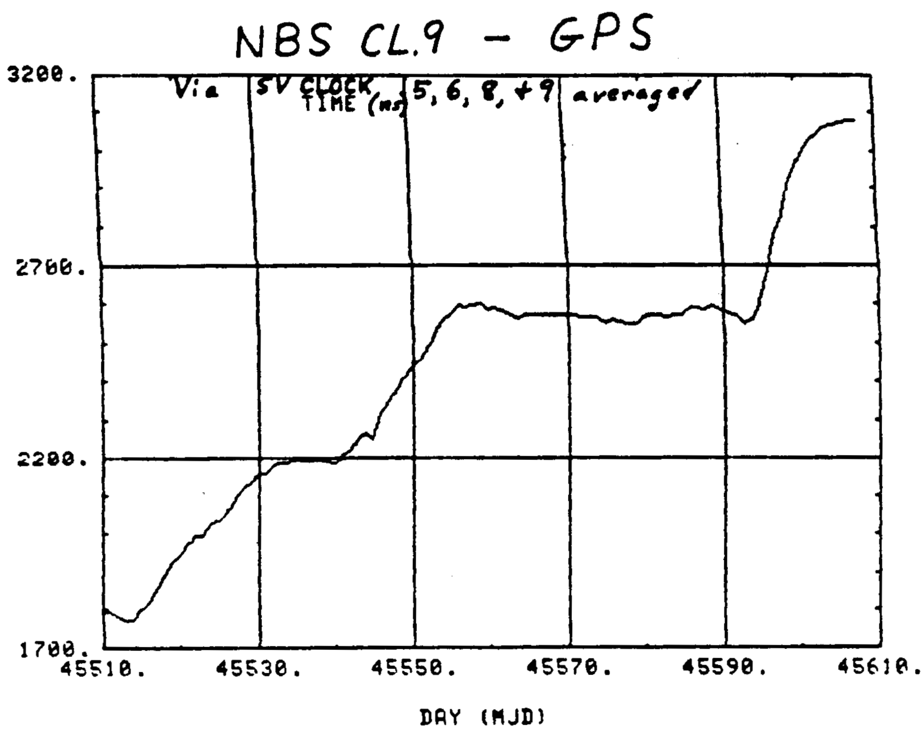
Figure 13 is a stability plot of Space Vehicle 9. In this case we have the reverse situation where the clock correction errors were very small and were not plotted. One explanation for this behavior is that too much error is being assigned to the ephemeris and propagation and not enough to the space vehicle clock. In long term, the space vehicle clock instability was sufficiently below the other instabilities in the system that it was unmeasurable and one gets some indication of its performance by the direct measurement against NBS shown earlier (Figure 8).

More recently, the separation of variance analysis technique was used to evaluate SV#11, the newest addition to the GPS constellation. This space vehicle is now operating with a rubidium standard. A linear least squares fit to the frequency of SV#11 of -2.65×10^{-13}/day frequency drift at about a 2% confidence of the estimate was removed from the data. After subtracting the frequency drift, the variances were separated. The frequency stability of the SV#11 rubidium at tau equal one day and longer was found to be excellent. In Figure 14, we show a comparison of the frequency stability of SV#11 with the SV#9 and 5 cesiums. One sees the very exciting result that at tau equal one and two days the SV#11 rubidium is comparable to the SV#9 cesium.

The software for separation of variance at NBS continues to undergo some refinements. However, the results to date are very enlightening concerning the performance of the GPS system and provide some insight into where one might improve the performance of the system.

[1] Jack Henrich, IBM Federal Systems Division, private communication.


UTC(USNO) — GPS
Via. SV CLOCK
TIME (ns)
1200.
700.
200.
-300.
45510.
45530.
45550.
45570.
45590.
45610.
DAY (MJD)
Fig. 1


NBS CL.9 — GPS
Via SV CLOCK
TIME (ns)
5, 6, 8, + 9 averaged
3200.
2700.
2200.
1700.
45510.
45530.
45550.
45570.
45590.
45610.
DAY (MJD)
Fig. 2

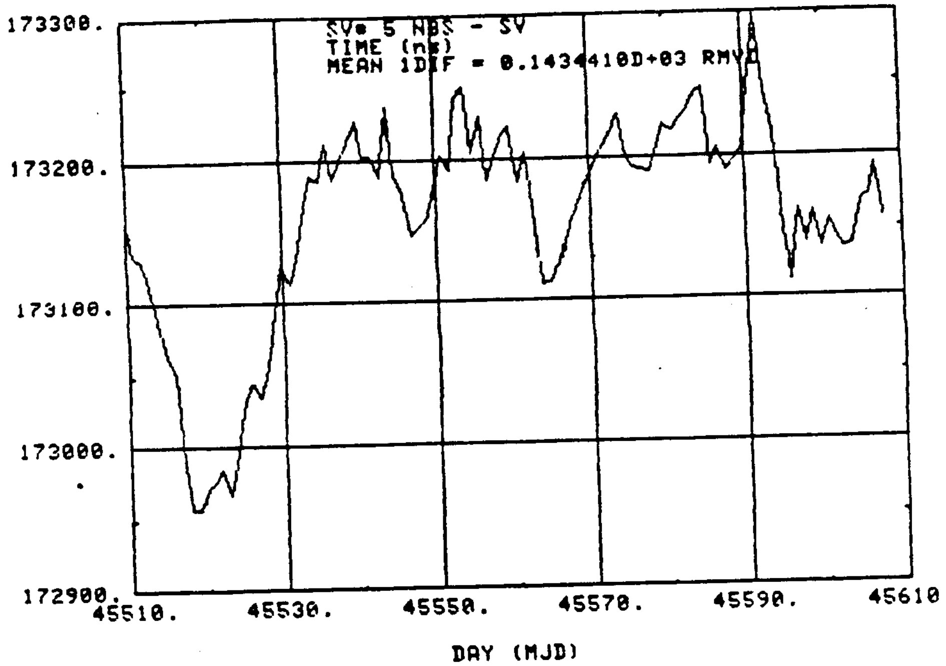

SV= 5 NBS - SV
TIME (ns)
MEAN 1DIF = 0.1434410D+03 RMYD
173300.
173200.
173100.
173000.
172900.
45510.
45530.
45550.
45570.
45590.
45610.
DAY (MJD)

Fig. 3

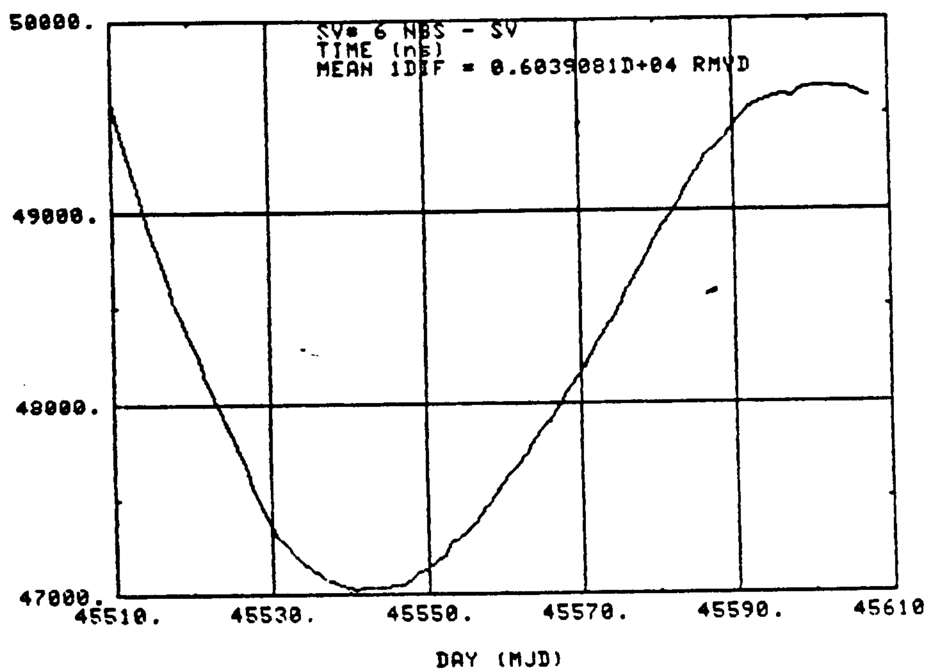

SV= 6 NBS - SV
TIME (ns)
MEAN 1DIF = 0.6039081D+04 RMYD
50000.
49000.
48000.
47000.
45510.
45530.
45550.
45570.
45590.
45610.
DAY (MJD)

Fig. 4

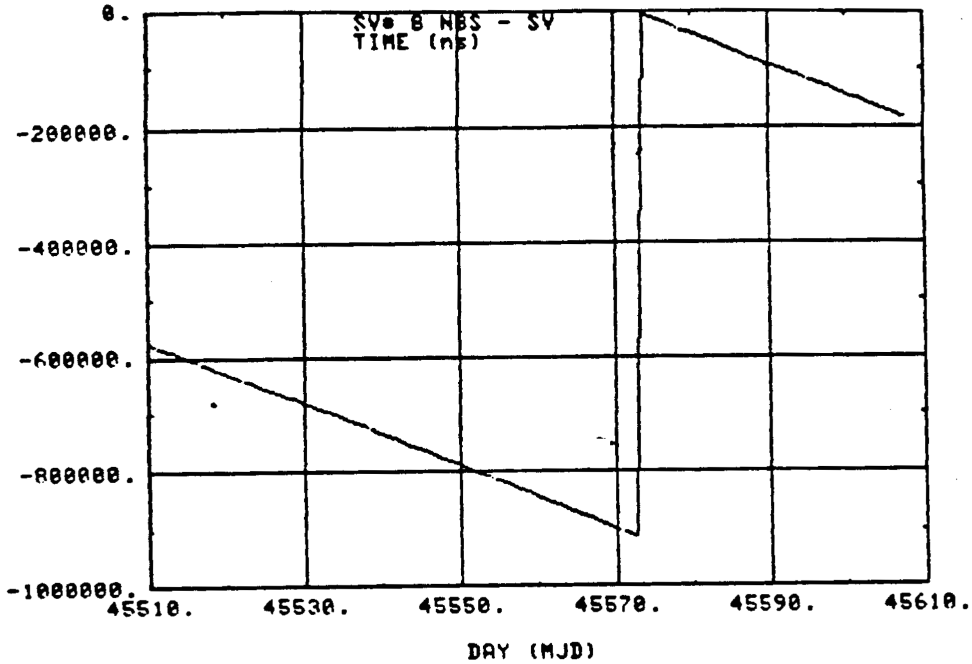

SV# 8 NBS - SV
TIME (ns)
0.
-200000.
-400000.
-600000.
-800000.
-1000000.
45510.
45530.
45550.
45570.
45590.
45610.
DAY (MJD)

Fig. 5

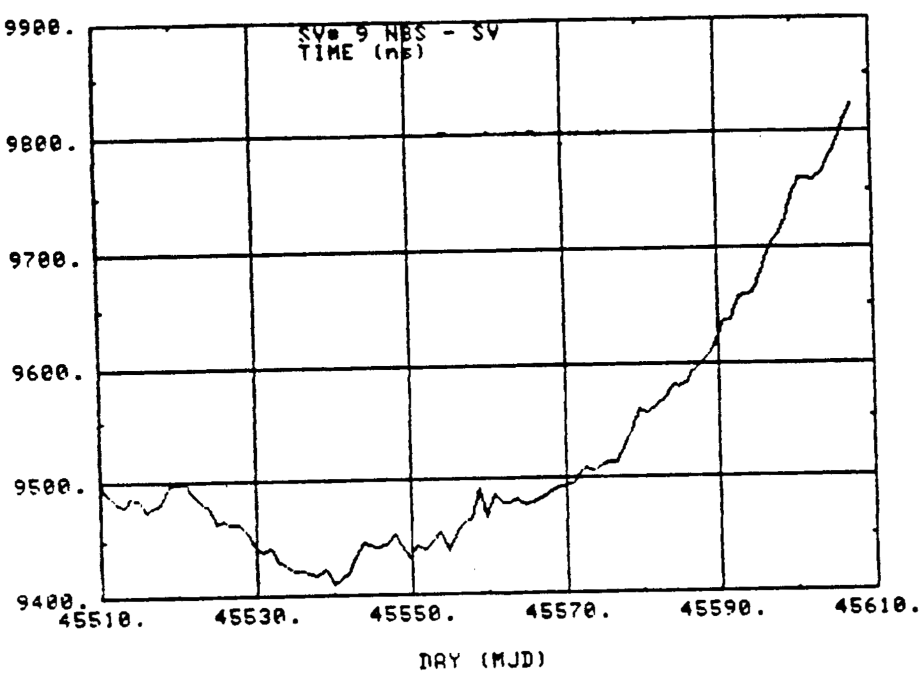

SV# 9 NBS - SV
TIME (ns)
9900.
9800.
9700.
9600.
9500.
9400.
45510.
45530.
45550.
45570.
45590.
45610.
DAY (MJD)

Fig. 6

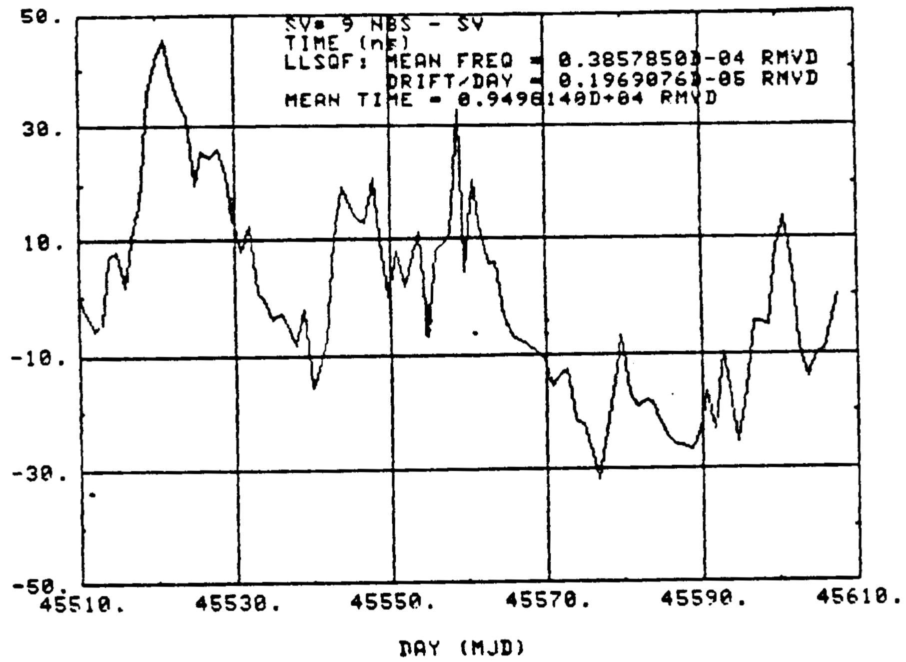

SV= 9 NBS - SV
TIME (nS)
LLSQF: MEAN FREQ = 0.385785OD-04 RMVD
DRIFT/DAY = 0.196907 6D-05 RMVD
MEAN TIME = 0.9498140D+04 RMVD
50.
30.
10.
-10.
-30.
-50.
45510.
45530.
45550.
45570.
45590.
45610.
DAY (MJD)

Fig. 7

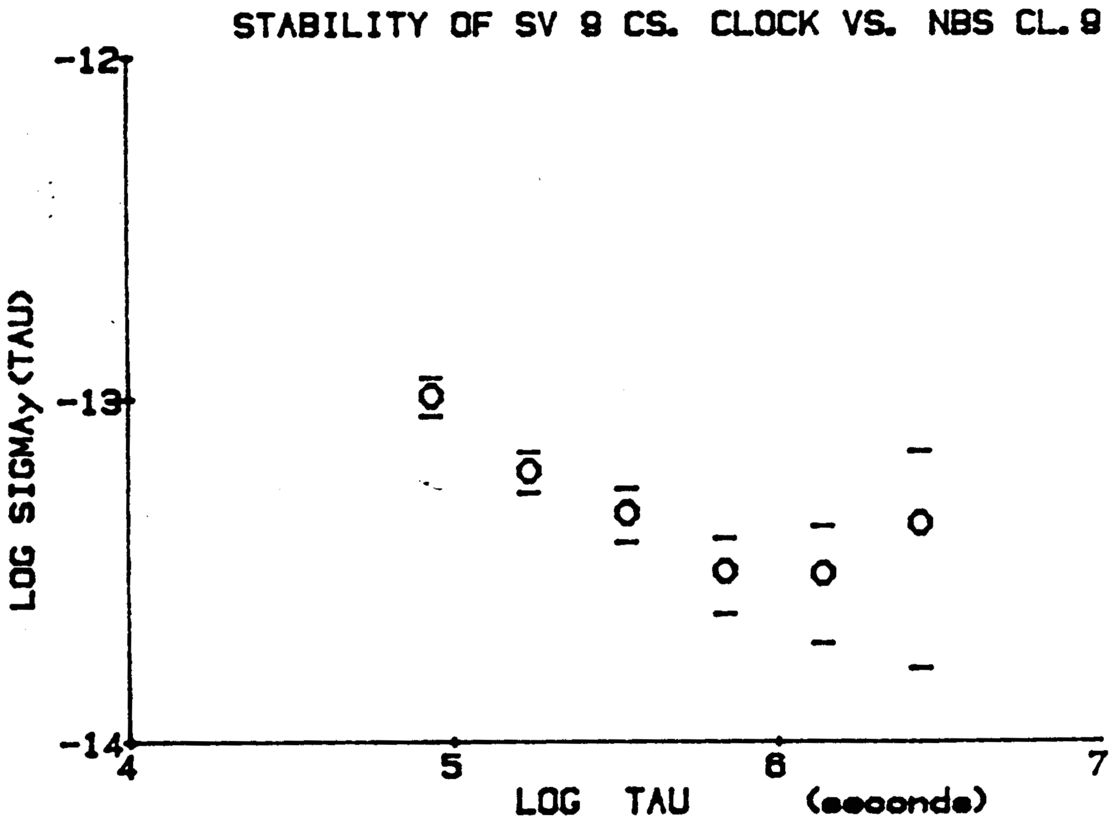
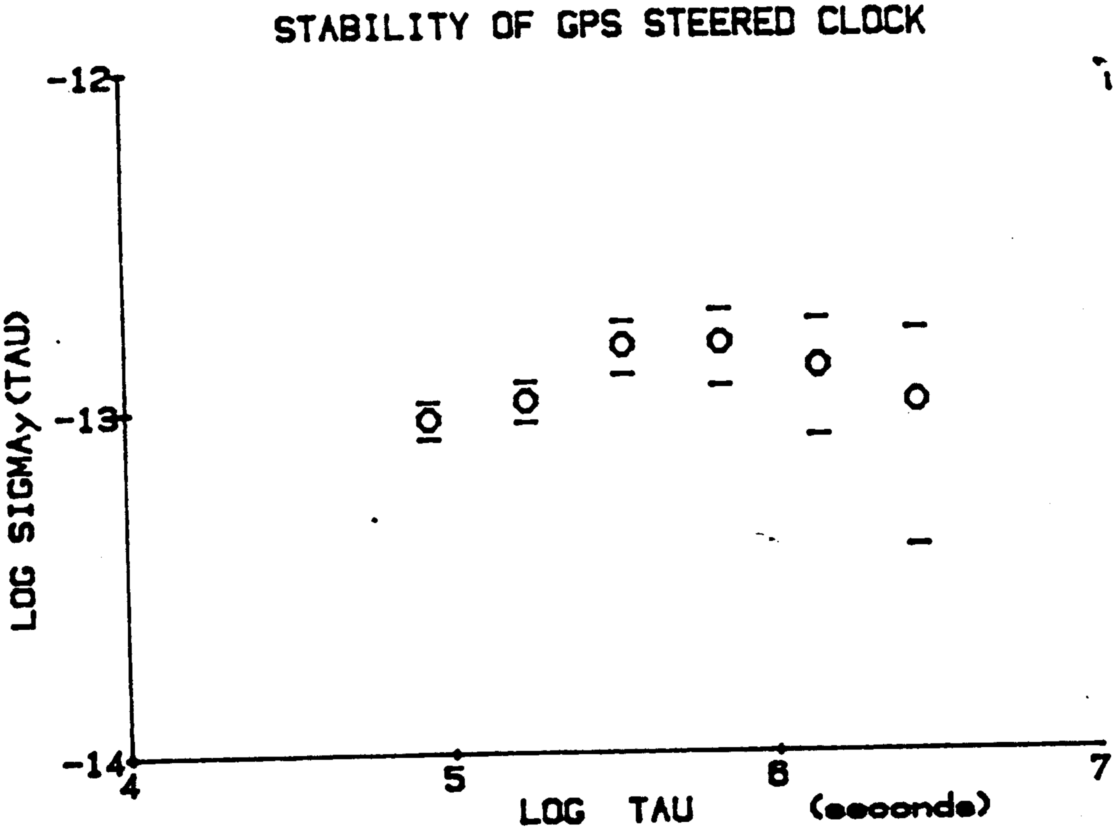
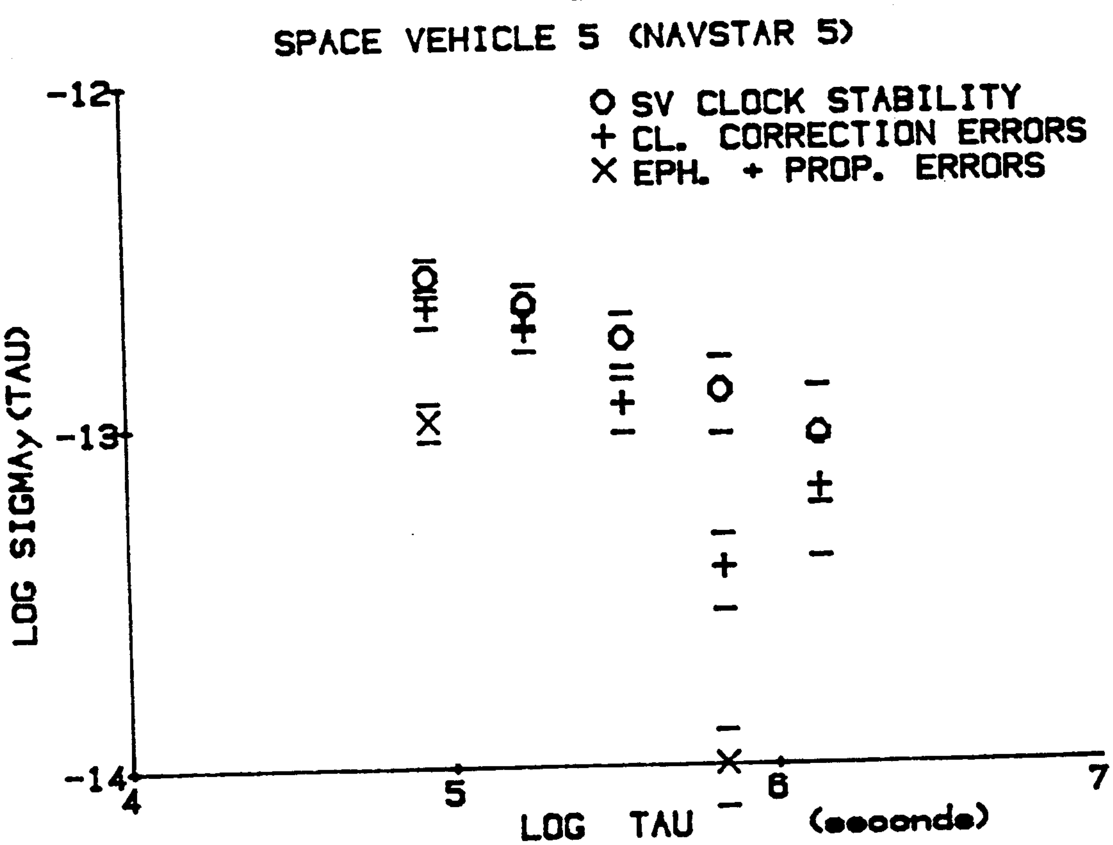

STABILITY OF SV 8 CS. CLOCK VS. NBS CL. 8
-12
-13
-14
LOG SIGMAy (TAU)
4
5
6
7
LOG TAU (seconds)

Fig. 8



STABILITY OF GPS STEERED CLOCK
-12
-13
-14
LOG SIGMAy (TAU)
4
5
6
7
LOG TAU (seconds)

Fig. 9



SPACE VEHICLE 5 (NAVSTAR 5)
O SV CLOCK STABILITY
+ CL. CORRECTION ERRORS
X EPH. + PROP. ERRORS
-12
-13
-14
LOG SIGMAy (TAU)
4
5
6
7
LOG TAU (seconds)

Fig. 10

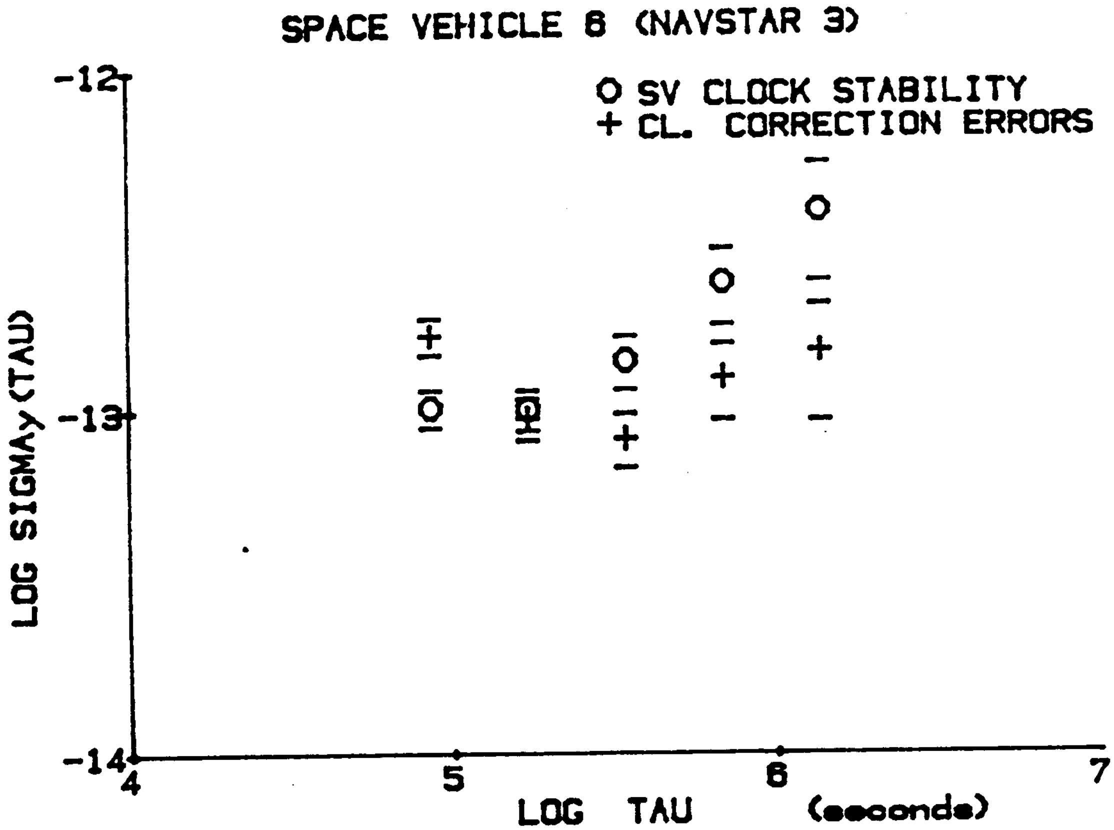

SPACE VEHICLE 6 (NAVSTAR 3)
O SV CLOCK STABILITY
+ CL. CORRECTION ERRORS
LOG SIGMAy(TAU)
-12
-13
-14
4
5
6
7
LOG TAU (seconds)

Fig. 11

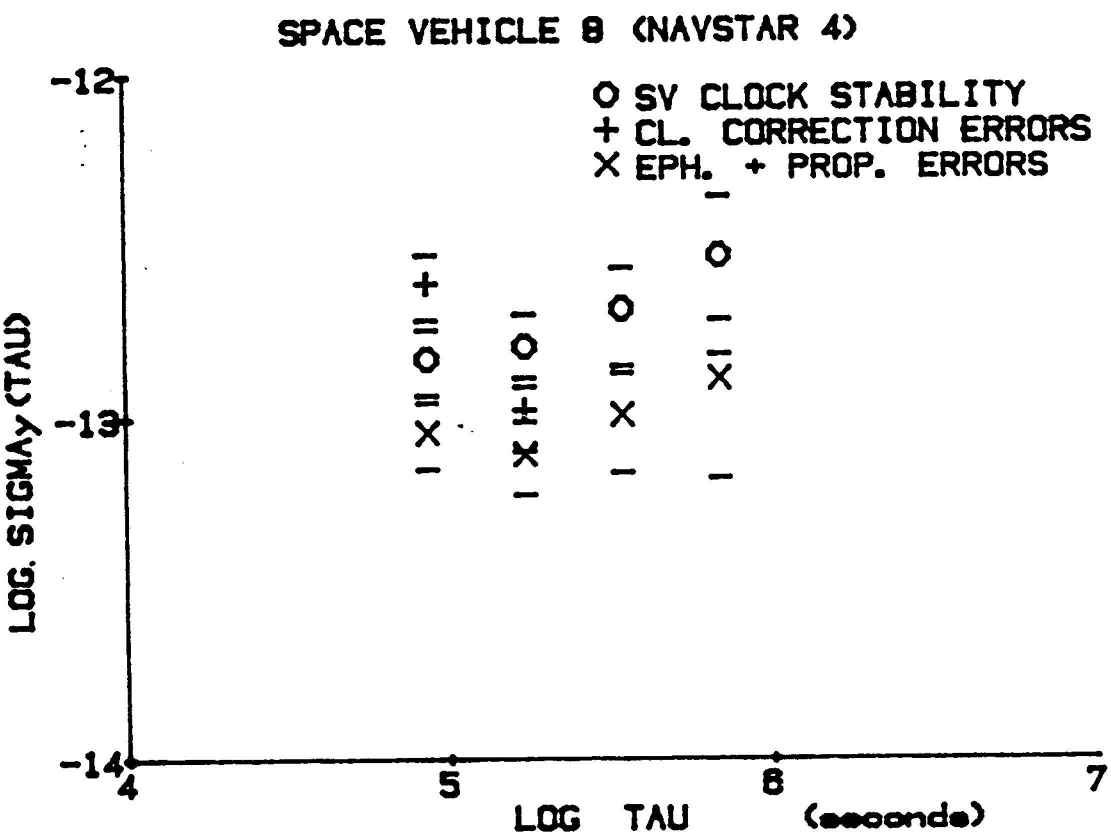

SPACE VEHICLE 8 (NAVSTAR 4)
O SV CLOCK STABILITY
+ CL. CORRECTION ERRORS
X EPH. + PROP. ERRORS
LOG SIGMAy(TAU)
-12
-13
-14
4
5
6
7
LOG TAU (seconds)

Fig. 12

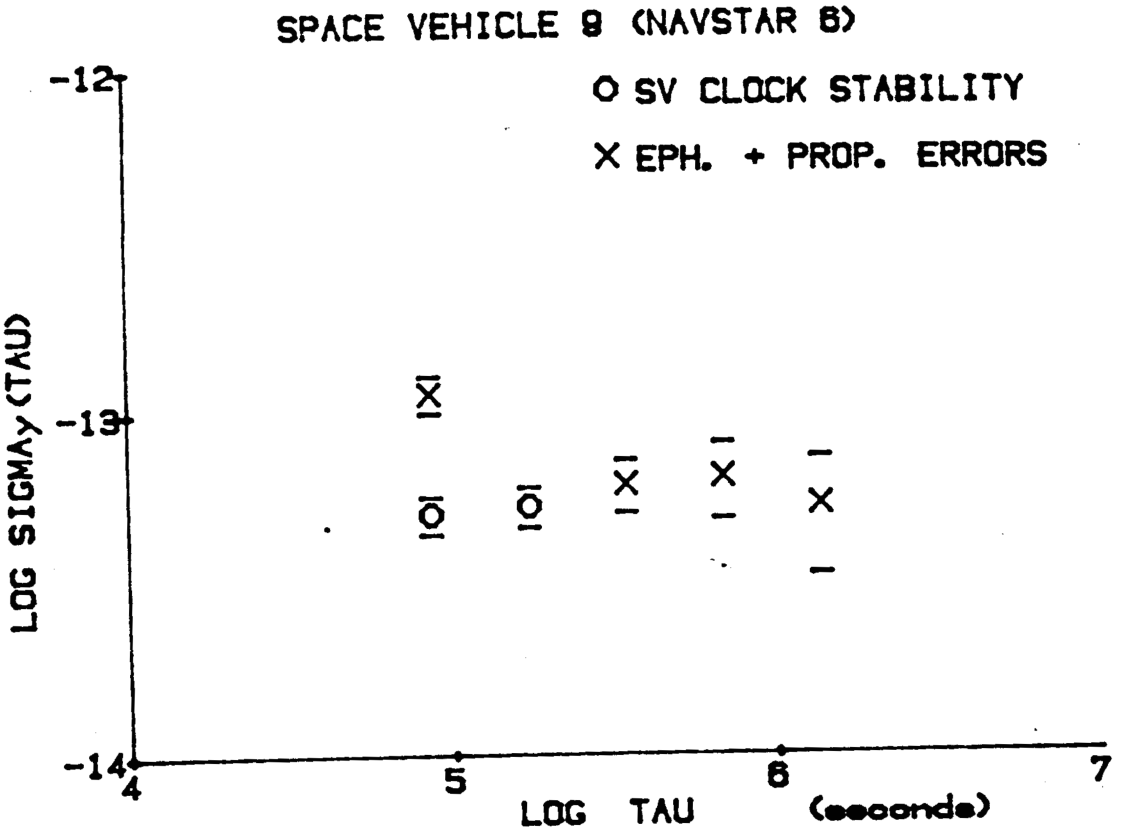

SPACE VEHICLE 8 (NAVSTAR 6)
O SV CLOCK STABILITY
X EPH. + PROP. ERRORS
LOG SIGMAy (TAU)
-12
-13
-14
4
5
6
7
LOG TAU (seconds)

Fig. 13

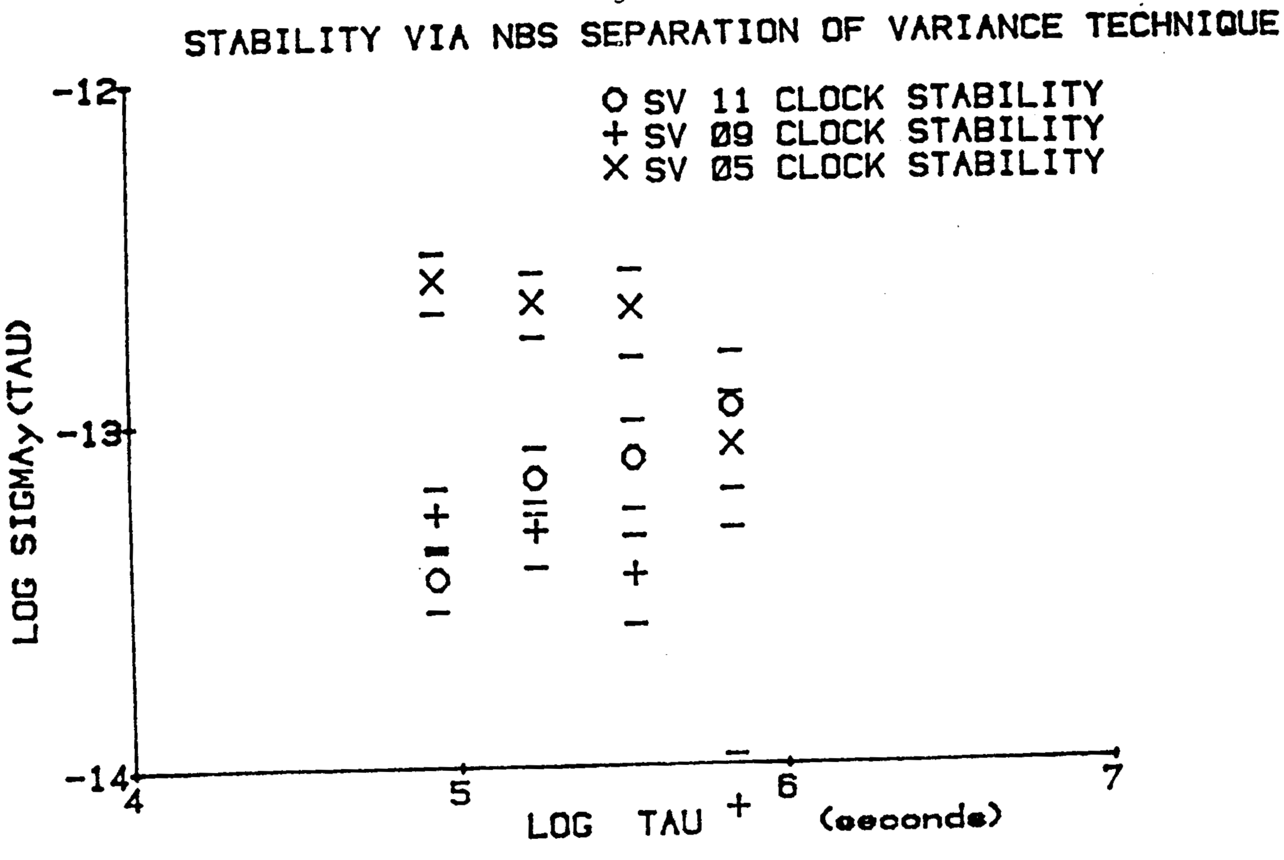

STABILITY VIA NBS SEPARATION OF VARIANCE TECHNIQUE
O SV 11 CLOCK STABILITY
+ SV Ø9 CLOCK STABILITY
X SV Ø5 CLOCK STABILITY
LOG SIGMAy (TAU)
-12
-13
-14
4
5
6
7
LOG TAU (seconds)

Fig. 14